



## BACHELOR IN ECONOMICS SECOND YEAR

<b>Course</b>	Mathematics III	<b>Code</b>	802353
<b>Module</b>	Quantitative Methods	<b>Area</b>	Advanced mathematics
<b>Nature</b>	Compulsory		
<b>Credits</b>	6	<b>Attendance</b>	3,3
		<b>Non Attendance</b>	2,7
<b>Year</b>	2	<b>Semester</b>	3

### COORDINATION

DEPARTMENT
Fundamentos del Análisis Económico I
COORDINATOR AND CONTACT
María Jesús Moreta; <a href="mailto:mjesusmoreta@ccee.ucm.es">mjesusmoreta@ccee.ucm.es</a>

### SYNOPSIS

BRIEF DESCRIPTION
<ol style="list-style-type: none"><li>1. Formulation, resolution and discussion of multivariable optimization problems with different types of constraints as the basic models for decision making in static economic environments.</li><li>2. Introduction to the basic theory of one and two dimensional dynamical systems and its role in modeling time-evolving economic phenomena.</li></ol>
PRE-REQUISITES
Mathematics I and II
OBJECTIVES
Formulate and solve prototype models of economic theory both static and dynamic as mathematical problems of optimization or dynamical equations, interpreting the obtained solutions and their properties from a careful mathematical analysis.



<b>COMPETENCES</b>
Generals: CG1, CG2, CG4 Transversals: CT1,CT2,CT3 Specifics: CE8, CE9
<b>LEARNING METHODOLOGY</b>
A mixed methodology of teaching and learning will be used in all educational activities with the aim of encouraging students to develop a collaborative and cooperative attitude in the pursuit of knowledge.
<b>TOPICS COVERED (Syllabus)</b>
<p>Unit 0. Introduction.</p> <p>Economics, derivatives and optimization. Static and dynamic problems. Solutions and their interpretation. Economic examples.</p> <p>PART 1: Static Optimization</p> <p>Unit 1. Multivariable optimization I (Basics)</p> <p>Local and global extrema. Extreme value theorem. Graphical method of solution. Convex sets. Concave functions. Quasiconcave functions.</p> <p>Global theorem of concave programming.</p> <p>Unit 2 Multivariable optimization II (Unconstrained problems)</p> <p>Necessary conditions for local interior optima. Saddle points.</p> <p>Second order conditions. Convex problems and global optima.</p> <p>Parametric problems. Comparative statics. Envelope theorem.</p> <p>Unit 3. Multivariable optimization III (Constrained problems).</p> <p>Problems with equality constraints. Lagrange necessary conditions Lagrange multipliers.</p> <p>General constrained problems. Sign of multipliers. Kuhn-Tucker conditions. Parametric problems. Envelope theorem.</p> <p>PART 2: Dynamical Systems</p> <p>Unit 4.-First-order equations in discrete time</p> <p>Difference equations. Autonomous equations. Solutions and their properties. Existence and uniqueness of solution.</p> <p>Linear case. Population growth. Loans and Mortgages Cobweb model.</p> <p>Unit 5.- First-order equations in continuous time</p> <p>Differential equations. Autonomous equations. Solutions and their properties. Existence and uniqueness of solution.</p> <p>Linear equation. Linear growth. Economic applications.</p> <p>Nonlinear equations. Logistic growth. Phase diagram. Equilibria: location and stability. Solow model.</p>



## Faculty of Economics and Business

Unit 6.- Dynamical problems in two variables.

Linear systems in two dimensions: continuous dynamics. Examples.

Equilibria and stability. Algebraic conditions for equilibrium stability.

Second-order dynamical equations as first-order dynamical systems.

TEACHING ACTIVITIES DISTRIBUTION		
Theoretical lessons	% of Total Credits	30%
Practical lessons	% of Total Credits	10%
In seminar sessions: one every other week.		
Other Activities	% of Total Credits	60%
Personalized & Group Tutoring: 15%, Assessment Activities, Individual & Group Homework Assignments, Hours to Study: 45%		
ASSESSMENT		
Exams	% Share of Final Grade	50 %
Final Exam 50%		
Other Activities	% Share of Final Grade	40%
Midterm Exam 30%		
Assigned problem sets (5/6) 10%		
Other activities	% Share of Final Grade	10%
Active participation in the classroom or in seminars and carrying out and presentation of individual or group projects 10%		
EVALUATION CRITERIA		
A minimum score of 3.5 in the final exam is required to pass the course.		
In the "convocatoria ordinaria" a student will be considered "presentado" if he/she fulfills the course requirements up to the point indicated by the professor. In the "convocatoria extraordinaria" a student will be considered "presentado" if he/she takes the final exam.		



## TIMETABLE

[15 weeks = 30 class sessions with 5/6 seminars]

Week/day	unit	class content	seminar content
1	0	Economics, derivatives and optimization. Static and dynamic problems. Solutions and their interpretations. Economic Examples.	
2	1	Local and global optima. Weierstrass Theorem. Properties of the gradient. The formula $D_v f(x_0) = \nabla f(x_0)^T v$ . Graphical method.	
3	1	Convex sets. Concave functions. Quasiconcave functions.	
4	2	Concave programming. SEMINAR	ProbSet #1. Optimization I
5	2	Necessary conditions for local extreme interior points. Saddle points. Second order conditions. Convex problems	
6	2	Problems depending on parameters. Comparative statics. Value function. Envelope theorem. Comparative statics of the competitive firm. SEMINAR	ProbSet #2: Optimization II
7	3	Problems with equality constraints. Solving by substitution. Lagrange necessary condition. Lagrange multipliers	
8	3	General constrained problems. Sign of multipliers. Parametric problems. Envelope theorem. SEMINAR	ProbSet #3: Optimization III
9	4	Difference equations. Autonomous equations. Solutions and their properties. Existence and uniqueness of solution. Linear case. Population growth	
10	4	Loans and Mortgages Cobweb model.. MIDTERM EXAM	I
11	4	Differential equations. Autonomous equations. Solutions and their properties. Existence and uniqueness of solution. SEMINAR	ProbSet #4: Dynamics I
12	5	Linear equation. Linear growth. Economic applications Nonlinear equations. Logistic growth. Phase diagram	
13	5	Equilibria: location and stability. Solow model Linear systems in two dimensions: continuous dynamics.	
14	6	Equilibria and stability. Algebraic conditions for stability. Dynamically invariant subspaces. Examples.	
15	6	Second-order dynamical equations as first-order dynamical systems SEMINAR	ProbSet #5: Dynamics II.



## RESOURCES

### BOOK LIST

K. Sydsaeter and P.J. Hammond (2008). *Essential Mathematics for Economic Analysis*. Prentice-Hall, 3<sup>rd</sup> ed.

C.P. Simon and L. Blume (1994). *Mathematics for economists*. WW Norton and Co.

K. Sydsaeter, P. Hammond, A. Seierstad, A. Strom, *Further Mathematics for Economic Analysis*, Prentice Hall, 2005.

Simon C.P. y Blume L. *Mathematics for Economist*. Norton 1994.

### OTHER RESOURCES